6389
9

RELAZIONI

FRA GLI ELEMENTI

DI UN

TRIANGOLO PIANO

NUM. 566 FORMULE

RACCOLTE ED ORDINATE

DAL

Prof. CRISTOFORO ALASIA

517

CITTÀ DI CASTELLO
S. LAPI, TIPOGRAFO-EDITORE

1900

RELAZIONI

FRA GLI ELEMENTI

DI UN

TRIANGOLO PIANO

NUM. 566 FORMULE

RACCOLTE ED ORDINATE

DAL

Prof. CRISTOFORO ALASIA

517

CITTÀ DI CASTELLO
S. LAPI, TIPOGRAFO-EDITORE

1900

m_a, m_b, m_c mediane condotte da A, B, C rispettivamente,
n_a, n_b, n_c insimediane condotte da A, B, C rispettivamente,
ν_a, ν_b, ν_c essimediane condotte da A, B, C rispettivamente,
O circoncentro di ABC,
O_9 centro del cerchio di FEUERBACH,
O_a, O_b, O_c circoncentri dei triangoli HCB, HCA, HAB rispettivamente,
O_0, O_1, O_2, O_3 circoncentri di $I_aI_bI_c$, II_bI_c, II_aI_c, II_aI_b,
r raggio dell'incerchio,
R raggio del circoncerchio,
R punto di STEINER,
R, S, T piedi delle insimediane,
R', S', T' piedi delle essimediane,
X, Y, Z piedi delle perpendicolari calate da A, B, C rispettivamente sul lato opposto,
Γ punto d'incontro di AD, BE, CF (punto di GERGONNE),
Γ_1, Γ_2, Γ_3 punti associati del punto di GERGONNE,
N punto di TARRY,
Ω, Ω' 1° e 2° punto di BROCARD,
ω angolo di BROCARD,
Z punto medio di OK (centro del cerchio di BROCARD),
S punto medio di $\Omega\Omega'$,
S' suo corrispondente. È all'intersezione delle rette che uniscono i punti medi dei lati di $A_1B_1C_1$ (1° triangolo di BROCARD) ai vertici di ABC,
D centro d'omologia di ABC e del 1° triangolo di BROCARD. È all'intersezione delle rette AA_1, BB_1, CC_1,
D' polo di $\Omega\Omega'$ rispetto al cerchio di BROCARD. È il punto medio di DH.
Δ area del triangolo ABC,
Δ_a, Δ_b, Δ_c area dei triangoli HCB, HCA, HAB.
Δ_0, Δ_1, Δ_2, Δ_3 area dei triangoli $I_aI_bI_c$, II_bI_c, II_aI_c, II_aI_b.

$$a^2 + b^2 + c^2 = m^2,$$
$$a^4 + b^4 + c^4 = q^4,$$
$$ab + bc + ca = t^2,$$
$$a^2b^2 + b^2c^2 + c^2a^2 = n^4,$$
$$b^2 + c^2 - a^2 = \alpha^2,$$
$$a^2 + c^2 - b^2 = \beta^2,$$
$$a^2 + b^2 - c^2 = \gamma^2.$$

Bisettrici.

1. $AL=\frac{2}{b+c}\sqrt{bcp\,(p-a)}$, $BM=\frac{2}{a+c}\sqrt{acp(p-b)}$, $CN=\frac{2}{a+b}\sqrt{abp(p-c)}$.

2. $AL=2\Delta\frac{\sqrt{bcrr_a}}{(b+c)rr_a}$, $BM=2\Delta\frac{\sqrt{acrr_b}}{(a+c)rr_b}$, $CN=2\Delta\frac{\sqrt{abrr_c}}{(a+b)rr_c}$.

3. $\overline{AL}^2=4rr_a\frac{rr_a+r_br_c}{(r+r_a)^2}$, ecc.

4. $AL'=\frac{2}{b-c}\sqrt{bc\,(p-b)\,(p-c)}$, $BM'=\frac{2}{a-c}\sqrt{ac\,(p-a)\,(p-c)}$,

$CN'=\frac{2}{a-b}\sqrt{ab\,(p-a)\,(p-b)}$.

5. $\overline{AL'}^2=4r_br_c\frac{rr_a+r_br_a}{(r_b-r_a)^2}$, ecc.

6. $AL'=\frac{2\Delta}{(b-c)r_br_c}\sqrt{bcr_br_c}$, $BM'=\frac{2\Delta}{(a-c)r_ar_c}\sqrt{acr_ar_c}$, $CN'=\frac{2\Delta}{(a-b)r_ar_b}\sqrt{abr_ar_b}$.

7. $MN=4\Delta\frac{OI_a}{(a+b)(a+c)}$, $NL=4\Delta\frac{OI_b}{(a+b)(b+c)}$, $LM=4\Delta\frac{OI_c}{(b+c)(c+a)}$.

8. $M'N'=4\Delta\frac{OI}{(a-b)(a-c)}$, $N'L'=4\Delta\frac{OI}{(a-b)(b-c)}$, $L'M'=4\Delta\frac{OI}{(a-c)(b-c)}$.

9. $L'M'N=\frac{l_c\lambda_a\lambda_b}{4(p-c)}$, $M'N'L=\frac{l_a\lambda_b\lambda_c}{4(p-a)}$, $L'N'M=\frac{l_b\lambda_a\lambda_c}{4(p-b)}$.

10. $LMN=\frac{l_al_bl_c}{4p}$, ecc.

11. $LMN=2\Delta\frac{abc}{(b+c)(c+a)(a+b)}$, ecc.

12. $M'N'L=2\Delta\frac{abc}{(b+c)(c+a)(a+b)}$, $L'N'M=2\Delta\frac{abc}{(b-c)(a-b)(a+c)}$.

13. $LL'M = 2\Delta \dfrac{abc}{(a+c)(b^2-c^2)}$, $\quad LL'N = 2\Delta \dfrac{abc}{(a+b)(b^2-c^2)}$.

14. $AE = 2\dfrac{bc^2}{m^2}$, $\quad BF = 2\dfrac{ca^2}{m^2}$, $\quad CD = 2\dfrac{ab^2}{m^2}$.

15. $AF = c\dfrac{n_a^2}{m^2}$, $\quad BD = a\dfrac{n_b^2}{m^2}$, $\quad CE = b\dfrac{n_c^2}{m^2}$.

16. $BL = \dfrac{ac}{b+c}$, $\quad CL = \dfrac{ab}{b+c}$.

17. $LD = \dfrac{(p-a)(b-c)}{b+c}$.

18. $A'L = \dfrac{a(b-c)}{2(b+c)}$, $\quad A'L' = \dfrac{a(b+c)}{2(b-c)}$.

19. $LD_1 = \dfrac{p(b-c)}{b+c}$, $\quad L'D_2 = \dfrac{(p-b)(b+c)}{b-c}$, $\quad L'D_3 = \dfrac{(p-b)(b+c)}{b-c}$.

20. $LL' = l_a^2 + \lambda_a^2$, $\quad MM' = l_b^2 + \lambda_b^2$, $\quad NN' = l_c^2 + \lambda_c^2$.

21. $l_a l_b l_c = \Delta \dfrac{8p \,.\, abc}{(a+b)(b+c)(c+a)}$.

22. $\lambda_a\lambda_b\lambda_c = \Delta \dfrac{8abcr}{(a-b)(a-c)(b-c)}$.

23. $l_a l_b l_c = \Delta^3 \dfrac{32R}{r(a+b)(a+c)(b+c)}$.

24. $l_a l_b l_c = \dfrac{8\Delta^2}{h_a + h_b + h_c - r}$.

25. $\lambda_a\lambda_b\lambda_c = \Delta \dfrac{32R}{p(a-b)(a-c)(b-c)}$.

26. $l_a l_b l_c \lambda_a\lambda_b\lambda_c = \Delta^3 \dfrac{64a^2b^2c^2}{(a^2-b^2)(a^2-c^2)(b^2-c^2)}$.

27. $l_a l_b l_c \lambda_a\lambda_b\lambda_c = \Delta^5 \dfrac{1024R^2}{(a^2-b^2)(a^2-c^2)(b^2-c^2)}$.

28. $l_a^2 = 4rr_a \dfrac{rr_a + r_b r_c}{(r-r_a)^2}$, ecc.

29. $\lambda_a^2 = 4r_b r_c \dfrac{rr_a + r_b r_c}{(r_b - r_c)^2}$, ecc.

30. $\dfrac{1}{l_a^2} + \dfrac{1}{\lambda_a^2} = \dfrac{1}{h_a^2}$, ecc.

NOTA. Alcune fra le formule che precedono e molte fra quelle che seguono sono ottenute mediante una *trasformazione* detta *continua*, studiata da E. LEMOINE. L'idea fondamentale di questa trasformazione è la seguente: supponiamo che la base BC del triangolo ABC resti fissa nel mentre che il vertice A si trasporta all'infinito da uno dei lati di BC per ritornare poi nella regione finita dal lato

opposto. Il triangolo fondamentale ABC si trasforma allora in un nuovo triangolo ABC, ed ogni grandezza relativa al triangolo primitivo si trasforma nel nuovo triangolo. Così gli elementi a, b, c, A, B, C si trasformano rispettivamente in a, $-b$, $-c$, $-A$, $\pi-B$, $\pi-C$: gli elementi r, r_a, r_b, r_c, R, Δ si trasformano rispettivamente in r_a, r, $-r_c$, $-r_b$, $-R$, $-\Delta$, ecc.

Ogni relazione fra alcune delle precedenti grandezze dà così luogo ad una nuova relazione. (Vegg. *Mathesis*, II, 1892).

Relazioni fra i raggi.

31. $R=\dfrac{(r_a+r_b)(r_ar_b+rr_c)}{4r_ar_b}$.

32. $R=\dfrac{(r_b+r_c)(r_br_c+rr_a)}{4r_br_c}$.

33. $R=\dfrac{abc}{4\sqrt{p(p-a)(p-b)(p-c)}}$.

34. $R=\dfrac{bc}{2h_a}$.

35. $R=\dfrac{2rp^2}{h_ah_b+h_bh_c+h_ah_c}$.

36. $r_ar_b+r_br_c+r_ar_c=p^2$.

37. $R=\dfrac{1}{4p^2}(r_a+r_b)(r_b+r_c)(r_c+r_a)$.

38. $r=\sqrt{\dfrac{(p-a)(p-b)(p-c)}{p}}$.

39. $r_a=\sqrt{\dfrac{p(p-b)(p-c)}{p-a}}$, ecc.

40. $r^3=\dfrac{abc}{(a+b+c)^3}\cdot h_ah_bh_c$.

41. $2r=\dfrac{\overline{AL}^2}{AX}\sqrt{\dfrac{\overline{AA'}^2-\overline{AX}^2}{\overline{AL}^2-\overline{AX}^2}}$.

42. $2Rr=\dfrac{abc}{2\Delta}\cdot\dfrac{2\Delta}{a+b+c}$, $\qquad 2Rr_a=\dfrac{abc}{2\Delta}\cdot\dfrac{2\Delta}{b+c-a}$.

43. $2Rr\ \dfrac{abc}{a+b+c}$.

44. $\dfrac{1}{r}=\dfrac{1}{r_a}+\dfrac{1}{r_b}+\dfrac{1}{r_c}=\dfrac{1}{h_a}+\dfrac{1}{h_b}+\dfrac{1}{h_c}$.

45. $r_a + r_b + r_c = 4R + r.$

46. $r^2 + r_a^2 + r_b^2 + r_c^2 = 4R^2 + \overline{HA}^2 + \overline{HB}^2 + \overline{HC}^2.$

47. $r \cdot r_a \cdot r_b \cdot r_c = \Delta^2 = \Delta \cdot pr.$

48. $r \cdot r_a \cdot r_b \cdot r_c = \Delta r_a (p-a) = \Delta r_b (p-b) = \Delta r_c (p-c).$

49. $r_a r_b + r_a r_c - r_b r_c = p(2a - p).$

50. $r_b r_c + r r_a = bc.$

51. $(r_b + r_c)(r_c + r_a)(r_a + r_b) = 4Rp^2.$

52. $(r_a + r_b)(r_a + r_c)(r_b - r)(r_c - r) = b^2c^2.$

53. $r_a^2 + r_b^2 + r_c^2 = (4R + r)^2 - 2p^2.$

54. $(r_a + r_b + r_c)^2 = r_a^2 + r_b^2 + r_c^2 + 2p^2.$

55. $\left(\frac{r_a r_b}{r r_c} + 1\right)(r_c - r) = 4R.$

56. $-\frac{1}{r_a} = \frac{1}{R + OI_a} + \frac{1}{R - OI_a}, \qquad -\frac{1}{r_b} = \frac{1}{R + OI_b} + \frac{1}{R + OI_b},$ ecc.

57. $R(r_a + r_b)(r_b + r_c)(r_c + r_a) = \Delta_0^2, \qquad R(r_b + r_c)(r_b - r)(r_c - r) = \Delta_1^2;$ ecc.

58. $\frac{a}{r_b r_c} + \frac{b}{r_a r_c} + \frac{c}{r_a r_b} = \frac{2}{\Delta}(2R - r).$

59. $\frac{r_b r_c}{r^3} + \frac{r r_b}{r_c^3} + \frac{r r_c}{r_b^3} = (p-a)^2 r_a \left(\frac{1}{r^4} + \frac{1}{r_b^4} + \frac{1}{r_c^4}\right).$

60. $\left(\frac{r r_c}{r_a r_b} + 1\right)(r_a + r_b) = 4R.$

61. $\left(\frac{r_a r_b}{r r_c} + 1\right)(r_c - r) = 4R.$

62. $(r_a - r)(r_b - r)(r_c - r) = 4Rr^2.$

63. $(r_b - r_c)(r_c - r_a)(r_a - r_b)\frac{r}{p} = (b-c)(c-a)(a-b).$

64. $(r_a + r)(r_b + r)(r_c + r)\frac{p}{r} = (b+c)(c+a)(a+b)$

65. $\frac{r_b r_c}{r^2} - \frac{r r_b}{r_c^2} - \frac{r r_c}{r_b^2} = \frac{1}{r_a^2}\left[(p-a)^2 + 12Rr_a\right].$

66. $r_a r_b + r_a r_c - r_b r_c = p(2a - p).$

67. $\frac{r_b r_c}{a} + \frac{r_c r_a}{b} + \frac{r_a r_b}{c} = \frac{p}{4Rr}(p^2 + r^2 - 8Rr).$

68. $r_a^3 + r_b^3 + r_c^2 = (4R + r)^3 - 12Rp^2.$

69. $\frac{1}{r_a^3} + \frac{1}{r_b^3} + \frac{1}{r_c^3} = \frac{1}{r^3} - \frac{12R}{\Delta^2}.$

70. $r_a r_b = p(p - c).$

71. $r r_c = (p-a)(p-b).$

72. $r r_a = (p-b)(p-c).$

73 $\frac{1}{r} - \frac{1}{r_a} = \frac{1}{h_a}.$

74. $\frac{1}{r^2}+\frac{1}{r_a^2}+\frac{1}{r_b^2}+\frac{1}{r_c^2}=\frac{1}{\Delta^2}\,m^2.$

75. $\sqrt{\frac{r_a r_b r_c}{r}}=p.$

76. $\frac{h_b+h_c}{r_a}+\frac{h_c+h_a}{r_b}+\frac{h_a+h_b}{r_c}=6.$

77. $\frac{r_a r_b r_c}{h_a h_b h_c}=\frac{r_a r_b+r_b r_c+r_c r_a}{h_a h_b+h_b h_c+h_c h_a}.$

78. $1-\frac{r}{r_a}\,;\,1-\frac{r}{r_b}\,;\,1-\frac{r}{r_c}=a\,;\,b\,;\,c.$

79. $Rr=\frac{abc}{4p}.$

80. $Rr_a=\frac{abc}{4(p-a)}.$

81. $\frac{1}{2p}\,(ar_a^2+br_b^2+cr_c^2)=2R(4R+r)-p^2.$

82. $\frac{1}{2p}\,(a^2r_a+b^2r_b+c^2r_c)=r^2(4R+r)+p^2(2R+3r).$

83. $\frac{1}{2p}\,(ar_a^3+br_b^3+cr_c^3)=2R(4R+r^2)-p^2(6R-r).$

84. $ar_br_c+br_ar_c+cr_ar_b=2\Delta(4R+r).$

85. $\frac{ar_c}{r^2}+\frac{brr_b}{r_a^2}+\frac{crr_c}{r_b^2}=\frac{2}{\Delta}\left[2R(p-a)^2+r_a(4R+r)^2\right].$

86. $a^2r_a+b^2r_b+c^2r_c=4p^2(R-r).$

87. $p^2-r(r_a+r_b+r_c)=\frac{1}{2}\,(a^2+b^2+c^2).$

88. $r(4R+r)=a(p-a)+(p-b)(p-c).$

89. $2r(r_a+r_b+r_c)=a(p-a)+b(p-b)+c(p-c).$

90. $8Rr+2r^2=a(p-a)+b(p-b)+c(p-c).$

91. $(p-a)^2+r_a(4R-r_a)=\frac{1}{2}\,(a^2+b^2+c^2).$

92. $p\,[p^2-6Rr-3r(4R+r)]=\frac{1}{2}\,(a^3+b^3+c^3).$

93. $\frac{1}{r^2}+\frac{1}{r_a^2}=\frac{1}{\Delta^2}\left(p^2+(p-a)^2\right).$

94. $\frac{1}{r_b^2}+\frac{1}{r_c^2}=\frac{1}{\Delta^2}\left[(p-b)^2+(p-c)^2\right].$

95. $\frac{1}{r^2}+\frac{1}{r_a^2}=\frac{1}{2\Delta^2}\left[(b+c)^2+a^2\right].$

96. $\frac{1}{r_b^2}+\frac{1}{r_c^2}=\frac{1}{2\Delta^2}\left[(b-c)^2+a^2\right].$

97. $(r_b^2 - r_c^2)(r_a^2 - r^2) = a^2(b^2 - c^2)$.

98. $(r + r_a)(r_b + r_c) = a(b + c)$.

99. $(r_a - r)(r_b - r_c) = a(b - c)$.

100. $2rr_c = a(p - a) + b(p - b) + c(p - c)$.

101. $2R + r_a = \frac{1}{2\Delta}\left[ap^2 - b(p - c)^2 - c(p - b)^2\right]$.

102. $R + r = \frac{1}{4\Delta}\left[a^2(p - a) + b^2(p - b) + c^2(p - c)\right]$.

103. $2R - r = \frac{1}{2\Delta}\left[a(p - a)^2 + b(p - b)^2 + c(p - c)^2\right]$.

104. $4R + r = \frac{1}{2\Delta}\left[ar_br_c + br_ar_c + cr_ar_b\right]$.

105. $4Rr^2 = (r_a - r)(r_b - r)(r_c - r)$.

106. $\frac{2}{\Delta}(2R + r_a) = \frac{a}{r_br_c} + \frac{b}{rr_b} + \frac{c}{rr_c}$.

107. $a^3r_a + b^3r_b + c^3r_c = 2p(4Rr^2 + r^3 + 2Rp^2 - 3rp^2)$.

108. $r_abc + r_bca + r_cab = r\left[(4R + r)^2 + p^2\right]$.

109. $cr_b - br_c = \frac{1}{r}(p - a)^2(b - c) = \frac{r_br_c}{r_a}(b - c)$.

110. $br_b - cr_c = \frac{p^2}{r_a}(b - c) = \frac{r_br_c}{r}(b - c)$.

111. $cr_b + br_c = \frac{pr}{r_a}(4R + r + r_a)$.

112. $br_b + cr_c = p(4R - r - r_a)$.

113. $a\frac{r_br_c}{r} - b\frac{rr_b}{r_c} - c\frac{rr_c}{r} = 2(p - a)(2R + r_a)$.

114. $\frac{1}{r_a - r} + \frac{1}{r_a + r_b} + \frac{1}{r_a + r_c} = \frac{(p - a)^2 + 8Rr_a + r_a^2}{4Rar^2}$.

115. $br_c + cr_b - ar = 2(p - a)(2R + r)$.

116. $ar_br_c = ap(r - a)$.

117. $r_ar_b + r_ar_c - r_br_c = p(2a - p)$.

118. $rr_c + rr_b + r_br_c = p^2 - a^2$.

119. $ar_a + br_b + cr_c = 2p(2R - r)$.

120. $cr = (p - c)(r_c - r)$.

121. $cr_c = p(r_c - r)$.

122. $a^2 + b^2 + c^2 + r^2 + r_a^2 + r_b^2 + r_c^2 = 16R^2$.

123. $a^2 + b^2 + c^2 = 2p^2 - 2r^2 - 8Rr$.

124. $p^2 + r^2 + 4Rr = a^2 + b^2 + c^2$.

125. $\Sigma\frac{a}{r_br_c} = \frac{2}{\Delta}(2R - r)$.

126. $\Sigma\, a^2 (p-a) = 4\,\Delta\,(R+r)$.

127. $\Sigma\,(p-a)^2 = p^2 - 12\,\Delta\,R$.

128. $\Sigma\, a^3 (b-c)^3 = 12\,\Delta\,R\,(a-b)\,(b-c)\,(c-a)$.

129. $\Sigma\,\dfrac{a r_a}{r_a - r} = 3\,p$.

130. $\Sigma\,\dfrac{a\,(r_a^2 + r^2)}{r_a - r} = 2\,p\,(2R + r)$.

131. $r_a + r_b + r_c = 3R + r\,(\cos A + \cos B + \cos C)$.

132. $r_a r_b r_c = abc \cos\dfrac{A}{2} \cos\dfrac{B}{2} \cos\dfrac{C}{2}$.

133. $r_a r_b r_c \operatorname{cotg}\dfrac{A}{2} \operatorname{cotg}\dfrac{B}{2} \operatorname{cotg}\dfrac{C}{2} = p^3$.

Formule riferentesi ai centri I, I_a, I_b, I_c, O, O_9.

134. $II_a = a\sqrt{\dfrac{bc}{p\,(p-a)}}$, $\quad II_b = b\sqrt{\dfrac{ac}{p\,(p-b)}}$, $\quad II_c = c\sqrt{\dfrac{ab}{p\,(p-c)}}$.

135. $I_aI_b = c\sqrt{\dfrac{ab}{(p-a)(p-b)}}$, $\quad I_aI_c = b\sqrt{\dfrac{ac}{(p-a)(p-c)}}$, $\quad I_bI_c = a\sqrt{\dfrac{bc}{(p-b)(p-c)}}$.

136. $\overline{II_a}^2 = 4R(r_a - r)$, $\quad \overline{II_b}^2 = 4R(r_b - r)$, $\quad \overline{II_c}^2 = 4R(r_c - r)$.

137. $\overline{I_aI_b}^2 + \overline{I_aI_c}^2 + \overline{I_bI_c}^2 = 8R(4R + r)$.

138. $\overline{II_a}^2 + \overline{II_b}^2 + \overline{II_c}^2 + \overline{I_aI_b}^2 + \overline{I_aI_c}^2 + \overline{I_bI_c}^2 = 48R^2$.

139. $\overline{OI}^2 = R^2 - 2Rr$.

140. $\overline{OI_a}^2 = R^2 + 2Rr_a$, ecc.

141. $\overline{OI}^2 + \overline{OI_a}^2 + \overline{OI_b}^2 + \overline{OI_c}^2 = 4R^2 + 2R\,(r_a + r_b + r_c - r)$.

142. $3\left(\overline{OI}^2 + \overline{OI_a}^2 + \overline{OI_b}^2 + \overline{OI_c}^2\right) = 4m^2 + 4\,.\,\overline{OH}^2$.

143. $OA' + OB' + OC' = R + r$.

144. $OB' + OC' - OA' = r_a - R$, $\quad OA' + OC' - OB' = r_b - R$, $\quad OA' + OB' - OC' = r_c - R$.

Distanze di O dai lati di ABC:

145. $d_a = \dfrac{a\,(m^2 - 2a^2)}{8\Delta}$, $\quad d_b = \dfrac{b\,(m^2 - 2b^2)}{8\Delta}$, $\quad d_c = \dfrac{c\,(m^2 - 2c^2)}{8\Delta}$.

Se I' è la proiezione di I su BC e B', C' le proiezioni di I' su AC e AB rispettivamente:

146. $II' = \dfrac{\Delta}{p}$.

147. $IB = \frac{\Delta}{p}\operatorname{cotg}\frac{B}{2}, \qquad IC = \frac{\Delta}{p}\operatorname{cotg}\frac{C}{2}.$

148. $IB' = \frac{2\Delta}{p}\cos^2\frac{C}{2}, \qquad IC' = \frac{2\Delta}{p}\cos^2\frac{C}{2}.$

149. $AI \cdot BI \cdot CI = 4Rr^2$

150. $AI_a \cdot BI_b \cdot CI_c = 4Rr_a^2.$

151. $II_a \cdot II_b \cdot II_c = 16R^2r.$

152. $I_aI_b \cdot I_bI_c \cdot I_cI_a = 16R^2p.$

Distanza di O da BC:

153. $d_a = \frac{1}{4}(r + r_a + r_b - r_c).$

154. $\overline{OI}^2 = R^2 - 2Rr.$

155. $\overline{OI_a}^2 = R^2 + 2Rr_a, \qquad \overline{OI_b}^2 = R^2 + 2Rr_b, \qquad \overline{OI_c}^2 = R^2 + 2Rr_c.$

156. $IA = \sqrt{bc\frac{p-a}{p}}, \qquad IB = \sqrt{ac\frac{p-b}{p}}, \qquad IC = \sqrt{ab\frac{p-c}{p}}.$

157. $I_aA = \sqrt{bc\frac{p}{p-a}}, \qquad I_bB = \sqrt{ac\frac{p}{p-b}}, \qquad I_cC = \sqrt{ab\frac{p}{p-c}}.$

Poichè è $\frac{p}{p-a} = \frac{r_a}{r}, \quad \frac{p-b}{p-c} = \frac{r_a}{r_b}$, è:

158. $IA = \sqrt{bc\frac{r}{r_a}}, \qquad IB = \sqrt{ac\frac{r}{r_b}}, \qquad IC = \sqrt{ab\frac{r}{r_c}}.$

159. $I_aA = \sqrt{bc\frac{r_a}{r}}, \qquad I_aB = \sqrt{ac\frac{r_a}{r_c}}, \qquad I_aC = \sqrt{ab\frac{r_a}{r_b}}.$

160. $I_bA = \sqrt{bc\frac{r_b}{r_c}}, \qquad I_bB = \sqrt{ca\frac{r_b}{r_a}}, \qquad I_bC = \sqrt{ab\frac{r_a}{r_b}}.$

161. $I_cA = \sqrt{bc\frac{r_c}{r_b}}, \qquad I_cB = \sqrt{ca\frac{r_c}{r_a}}, \qquad I_cC = \sqrt{ab\frac{r_b}{r_c}}.$

162. $IL = \frac{a}{b+c}\sqrt{bc\frac{p-c}{p}}, \quad IM = \frac{b}{a+c}\sqrt{ac\frac{p-b}{p}}, \quad IN = \frac{c}{a+b}\sqrt{bc\frac{p-c}{p}}.$

163. $I_aL = \frac{a}{b+c}\sqrt{bc\frac{p-c}{p}}, \quad I_bM = \frac{b}{a+c}\sqrt{ac\frac{p}{p-b}}, \quad I_cN = \frac{a}{a+b}\sqrt{ab\frac{p}{p-c}}.$

164. $I_aB = \sqrt{ac\frac{p-c}{p-a}}$, ecc.

165. $I_bA = \sqrt{bc\dfrac{p-c}{p-b}}$, ecc.

166. $I_bL' = \dfrac{a}{b-c}\sqrt{bc\dfrac{p-c}{p-b}}$, $I_cL' = \dfrac{a}{b-c}\sqrt{bc\dfrac{p-b}{p-c}}$, ecc.

167. $IL = a\dfrac{\sqrt{bcrr_a}}{r_a(b+c)}$, $I_aL = a\dfrac{\sqrt{bcrr_a}}{r(b+c)}$, ecc.

168. $IL \cdot IM \cdot IN = \Delta\dfrac{16R^2r^2}{(a+b)(b+c)(c+a)}$, ecc.

169. $I_aL \cdot I_bM \cdot I_cN = \Delta\dfrac{16R^2p^2}{(a+b)(b+c)(c+a)}$, ecc.

170. $I_cL \cdot I_bM \cdot I_cN = \dfrac{4Rrp^2}{h_a+h_b+h_c-r}$.

171. $\dfrac{\overline{AI}^2}{bc} + \dfrac{\overline{BI}^2}{ca} + \dfrac{\overline{CI}^2}{ab} = 1.$

172. $\dfrac{bc}{\overline{AI_a}^2} + \dfrac{ca}{\overline{BI_b}^2} + \dfrac{ab}{\overline{CI_c}^2} = 1.$

173. $\overline{AI}^2 + \overline{BI}^2 + \overline{CI}^2 = ab + bc + ca - 12Rr.$

174. $AI \cdot BI \cdot CI = 4Rr^2.$

175. $O_0I = \dfrac{R}{2} - r.$

176. $O_0I_a = \dfrac{R}{2} + r_a$, $O_0I_b = \dfrac{R}{2} + r_b$, $O_0I_c = \dfrac{R}{2} + r_c.$

Distanza di O_0 dai lati di ABC:

177. $d_0 = r\cos A.$

Distanza di O_0 dal vertice A:

178. $d_0^2 = \dfrac{R^2+\alpha^2}{4}$.

179. $OBC = \dfrac{a^2}{16\Delta}\alpha^2$, $OCA = \dfrac{b^2}{16\Delta}\beta^2$, $OAB = \dfrac{c^2}{16\Delta}\gamma^2.$

180. $I_aBC = \dfrac{1}{2}ar_a$, $I_bAC = \dfrac{1}{2}br_b$, $I_cAB = \dfrac{1}{2}cr_c.$

181. $I_aBC = \dfrac{r^2_a(r_b+r_c)}{2\sqrt{r_br_c+r_cr_a+r_ar_b}}$, ecc.

182. $D_1E_1F_1 + D_2E_2F_2 + D_3E_3F_3 - DEF = 2\Delta.$

183. $DEF \cdot D_1E_1F_1 \cdot D_2E_2F_2 \cdot D_3E_3F_3 = \dfrac{\Delta^4}{16R^4}$.

184. $AI = \dfrac{p \operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$.

185. $AI = 2a\,\dfrac{\operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}}{\operatorname{sen} A}$.

186. $AI = 4R \operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}$.

187. $AI_a = \dfrac{p}{\cos\frac{A}{2}}$.

188. $II_a = \dfrac{a}{\cos\frac{A}{2}}$.

189. $II_a = 4R \operatorname{sen}\frac{A}{2}$.

190. $I_bI_c = \dfrac{a}{\operatorname{sen}\frac{A}{2}} = 4R\cos\frac{A}{2}$.

191. $\overline{OH}^2 = \dfrac{D^2}{4}(1 - 8\cos A\cos B\cos C)$.

192. $\overline{OH}^2 = \dfrac{9a^2b^2c^2 - m^2(2n^4 - p^4)}{2n^4 - p^4}$.

193. $\overline{OH}^2 = \dfrac{b^2}{4\operatorname{sen}^2 B}\left[(8\cos B\cos C - \operatorname{sen} B\operatorname{sen} C)^2 + \operatorname{sen}^2(B - C)\right]$.

Distanza di O dai lati di ABC:

194. $d_0 = \dfrac{a}{2}\operatorname{cotg} A$.

Distanza di H dai lati del triangolo stesso:

195. $d_h = \dfrac{a}{\operatorname{sen} A}\cos B\cos C$.

196. $d_0 \cdot d_h = \dfrac{D^2}{2}\cos A\cos B\cos C$.

197. $O_9I + O_9I_a + O_9I_b + O_9I_c = 6R$.

198. $\overline{O_9I}^2 + \overline{O_9I_a}^2 + \overline{O_9I_b}^2 + \overline{O_9I_c} = 2R^2 - m^2$.

199. $IOH = \dfrac{1}{8r}(a - b)(b - c)(c - a)$.

200. $I_aOH = \dfrac{1}{8r_a}(b - c)(c + a)(a + b)$.

Formule riferentisi alle mediane, baricentro, ecc.

201. $\overline{OG}^2 = \frac{1}{9}\left[9R^2 - 2\left(p^2 - 2(4R + r)\right)\right]$.

202. $\overline{OG}^2 = \frac{1}{9}\left[9R^2 - 2\left(p^2 - 2(r_a + r_b + r_c)\right)\right]$.

203. $\overline{OG}^2 = R^2 - \frac{2}{9}\left(p^2 - r(4R + r)\right)$.

204. $\overline{OG}^2 = R^2 - \frac{1}{2}m^2$.

205. $\overline{IG}^2 = \frac{1}{9}(p^2 + r^2 - 16Rr)$.

206. $\overline{IG}^2 = \frac{1}{9}\left[(p-a)^2 + 5r_a^2 + 16Rr_a\right]$.

207. $\overline{IG}^2 = \frac{5}{36}m^2 - \frac{1}{6}t^2 + r^2$.

208. $\overline{IG}^2 + \overline{I_aG}^2 + \overline{I_bG}^2 + \overline{I_cG}^2 = 16R^2 - \frac{4}{9}m^2$.

Distanze di G dai tre lati di ABC:

209. $d_a = \frac{2\Delta}{3a}, \quad d_b = \frac{2\Delta}{3b}, \quad d_c = \frac{2\Delta}{3c}$.

210. $3\left(\overline{AG}^2 + \overline{BG}^2 + \overline{CG}^2\right) = m^2$.

211. $12\left(\overline{A'G}^2 + \overline{B'G}^2 + \overline{C'G}^2\right) = m^2$.

212. $m_a \cdot AG + m_b \cdot BG + m_c \cdot CG = \frac{1}{2}m^2$.

213. $m_a \cdot A'G + m_b \cdot B'G + m_c \cdot C'G = \frac{1}{4}m^2$.

214. $8\left(m_b^2 + m_c^2 - \frac{1}{2}m_a^2\right) = 9a^2, \quad 8\left(m_a^2 + m_c^2 - \frac{1}{2}m_b^2\right) = 9b^2,$

$8\left(m_a^2 + m_b^2 - \frac{1}{2}m_c^2\right) = 9c^2$.

Porzione m'_a di mediana m_a compresa fra le altezze corrispondenti a B e C:

215. $m'_a = 4m_a \frac{m_a^2(b^2 - c^2)}{(3b^2 + c^2 - a^2)(3c^2 + b^2 - a^2)}$.

216. $m'_a = 2m_a \frac{(b^2 - c^2)\cos A}{(b + c\cos A)(c + b\cos A)}$.

Porzione m'_a di mediana m_a compresa fra le bisettrici di $\widehat{B}$ e $\widehat{C}$:

217. $m'_a = (b-c)\,\dfrac{2\,am_a}{(2\,c+a)\,(2\,b+a)}$.

218. $G_aG_bG_c = \dfrac{4}{9}\,\Delta^3\,\dfrac{m^2}{a^2b^2c^2}$.

219. $GG_bG_c = \dfrac{4}{9}\cdot\dfrac{\Delta^3}{b^2c^2}$, $\quad GG_aG_c = \dfrac{4}{9}\cdot\dfrac{\Delta^3}{a^2c^2}$, ecc.

220. $IG = \dfrac{1}{3}\,OI$.

221. $OG = 2\,.\,GI$.

222. $IG = \dfrac{1}{6}\,OH$.

223. $BG_a = \dfrac{3\,a^2 - b^2 + c^2}{6\,a}$, $\quad CG_b = \dfrac{3\,b^2 + a^2 - c^2}{6\,b}$, $\quad AG_c = \dfrac{3\,c^2 + b^2 - a^2}{6\,c}$.

224. $CG_a = \dfrac{3\,a^2 + b^2 - c^2}{6\,a}$, $\quad AG_b = \dfrac{3\,b^2 + c^2 - a^2}{6\,b}$, $\quad BG_c = \dfrac{3\,c^2 + a^2 - b^2}{6\,c}$.

225. $G_bG_c = \Delta\,\dfrac{4\,m_a}{3\,bc}$, $\quad G_aG_c = \Delta\,\dfrac{4\,m_b}{3\,ca}$, $\quad G_aG_b = \Delta\,\dfrac{4\,m_c}{3\,ab}$.

226. $m_a = \dfrac{bc}{R}$, $\quad m_b = \dfrac{ac}{R}$, $\quad m_c = \dfrac{ab}{R}$.

227. $m_a = \dfrac{1}{2}\sqrt{2\,(b^2+c^2) - a^2}$.

228. $m_a^2 + m_b^2 + m_c^2 = \dfrac{3}{4}\,m^2$.

229. $4\,m_a^2 = 2\,(b^2+c^2) - a^2$, $\quad 4\,m_b^2 = 2\,(a^2+c^2) - b^2$, $\quad 4\,m_c^2 = 2\,(a^2+b^2) - c^2$.

230. $4\,(m_a^2 + m_b^2 + m_c^2) = 3\,m^2$, ecc.

231. $16\,(m_a^4 + m_b^4 + m_c^4) = 9\,(a^4 + b^4 + c^4)$.

232. $4\,m_a^2 = (b+c)^2 + (b-c)^2 - a^2$, ecc.

233. $AX = \dfrac{2}{a}\sqrt{p\,(p-a)\,(p-b)\,(p-c)}$, ecc.

234. $\left(AX + BY + CZ\right)\left(\dfrac{1}{AX} + \dfrac{1}{BY} + \dfrac{1}{CZ}\right) = 2\,p\left(\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}\right)$.

235. $AX + BY + CZ = \dfrac{1}{2\,R}\,t^2$.

236. $CX = \dfrac{a^2}{2\,a}$, ecc.

237. $ABX = \dfrac{\gamma^2\,\Delta}{2\,a^2}$, $\quad BCY = \dfrac{\gamma^2\,\Delta}{2\,b^2}$, $\quad CAZ = \dfrac{\alpha^2\,\Delta}{2\,c^2}$.

238. $AYZ = \Delta \frac{\alpha^4}{4 b^2 c^2}$; $BXZ = \Delta \frac{\beta^4}{4 a^2 c^2}$, $CXY = \Delta \frac{\gamma^4}{4 a^2 b^2}$.

239. $XYZ = \Delta \frac{\alpha^2 \cdot \beta^2 \cdot \gamma^2}{4 a^2 b^2 c^2}$.

Porzione h'_a di altezza compresa fra le bisettrici di $\widehat{B}$ e $\widehat{C}$:

240. $h'_a = pr \frac{c - b}{r_a^2}$.

241. $h'_a = \frac{(c - b)(p - a)}{r_a}$.

242. $\overline{HI}^2 = 4(R^2 + 2Rr) + t^2 - m^2$.

243. $\overline{HI}^2 = (2R + r)^2 = 2r^2 - p^2$.

244. $\overline{HI}^2 = 4(R^2 - Rr) + 3r^2 - p^2$.

245. $\overline{HI_a}^2 = 4(R^2 + 2Rr_a) + bc - ca - ab - m^2$.

246. $\overline{HI_b}^2 = 4(R^2 + 2Rr_b) - bc + ca - ab - m^2$.

247. $\overline{HI_c}^2 = 4(R^2 + 2Rr_c) - bc - ca + ab - m^2$.

248. $\overline{HO}^2 = 9R^2 - 2\left(p^2 - r(4R + r)\right)$.

Distanza di H dai tre lati di ABC:

249. $d_a = \frac{1}{8\Delta a}(m^2 - 2b^2)(m^2 - 2c^2)$, $d_b = \frac{1}{8\Delta b}(m^2 - 2a^2)(m^2 - 2c^2)$,

$d_c = \frac{1}{8\Delta c}(m^2 - 2a^2)(m^2 - 2b^2)$.

250. $HBC = \frac{\beta^2 \cdot \gamma^2}{16\Delta}$, $HAC = \frac{\alpha^2 \cdot \gamma^2}{16\Delta}$, $HAB = \frac{\alpha^2 \cdot \beta^2}{16\Delta}$.

251. $HBX = \frac{\alpha^4 \cdot \gamma^4}{32 a^2 \Delta}$, $HCX = \frac{\beta^4 \cdot \gamma^4}{32 a^2 \Delta}$, ecc.

Centri isogonici.

252. $AV + BV + CV = \sqrt{\frac{m^2}{2} + 2\Delta\sqrt{3}}$.

253. $\overline{AV}^2 + \overline{BV}^2 + \overline{CV}^2 = \frac{m^2}{2} - \frac{2\Delta}{\sqrt{3}}$.

254. $\overline{AV}^2 + \overline{BV}^2 + \overline{CV}^2 = \frac{m^2}{2} - \frac{abc}{2R\sqrt{3}}$.

255. $\overline{AV'}^2 + \overline{BV'}^2 + \overline{CV'}^2 = \frac{m^2}{2} + \frac{abc}{2R\sqrt{3}}$.

Simediane, punto di Lemoine, ecc.

256. $BR = \frac{ac^2}{b^2+c^2}$, $CS = \frac{ba^2}{c^2+a^2}$, $AT = \frac{cb^2}{a^2+b^2}$.

257. $CR = \frac{ab^2}{b^2+c^2}$, $AS = \frac{bc^2}{c^2+a^2}$, $BT = \frac{ca^2}{a^2+b^2}$.

258. $AR = m_a \frac{2bc}{b^2+c^2}$, $BS = m_b \frac{2ca}{c^2+a^2}$, $CT = m_c \frac{2ab}{a^2+b^2}$.

259. $BR' = \frac{ac^2}{c^2-b^2}$, $CS' = \frac{ba^2}{c^2-a^2}$, $AT' = \frac{cb^2}{b^2-a^2}$.

260. $AR' = \frac{abc}{c^2-b^2}$, $BS' = \frac{abc}{c^2-a^2}$, $CT' = \frac{abc}{b^2-a^2}$.

261. $RR' = 2\frac{ab^2c^2}{c^4-b^4}$, $SS' = 2\frac{bc^2a^2}{c^4-a^4}$, $TT' = 2\frac{cb^2a^2}{b^4-a^4}$.

Distanze di K dai tre lati di ABC:

262. $d_a = 2a\frac{\Delta}{m^2}$, $d_b = 2b\frac{\Delta}{m^2}$, $d_c = 2c\frac{\Delta}{m^2}$.

263. $AK = 2bc\frac{\alpha m_a}{m^2}$, $BK = 2ac\frac{\beta m_b}{m^2}$, $CK = 2ab\frac{\gamma m_c}{m^2}$.

264. $RK = 2a^2bc\frac{\alpha n_a}{m^2(b^2+c^2)}$, $SK = 2b^2ac\frac{\beta n_b}{m^2(c^2+a^2)}$, $TK = 2c^2ab\frac{\gamma n_c}{m^2(a^2+\beta^2)}$.

265. $AK_1 = 2bc\frac{m_a}{\alpha^2}$, e valori analoghi per BK_2, CK_3.

266. $RK_1 = 2a^2bc\frac{m_a}{\alpha^2(b^2+c^2)}$, e valori analoghi per SK_2, TK_3.

267. $KK_1 = 4a^2bc\frac{m_a}{\alpha^2 m^2}$, e valori analogbi per KK_2, KK_3.

268. $K_2K_3 = 2bc\frac{a}{\beta^2+\gamma^2}$, e valori analoghi per K_3K_1, K_1K_2.

269. $K_bK_c = 4\Delta\frac{m_a}{m^2}$, $K_aK_c = 4\Delta\frac{m_b}{m^2}$, $K_aK_b = 4\Delta\frac{m_c}{m^2}$.

270. $BK_a = \frac{a}{2m^2}(a^2-b^2+3c^2)$, $AK_b = \frac{b}{2m^2}(b^2-a^2+3c^2)$, $CK_c = \frac{a}{2m^2}(a^2-c^2+3b^2)$.

271. $CK_c = \frac{b}{2m^2}(b^2-c^2+3a^2)$, $AK_c = \frac{c}{2m^2}(c^2-a^2+3b^2)$, $BK_c = \frac{c}{2m^2}(b^2-c^2+3a^2)$.

272. $K_1K'_a = \Delta\frac{-2a}{\alpha^2}$, $K_1K'_b = \Delta\frac{2b}{\alpha^2}$, $K_1K'_c = \Delta\frac{2c}{\alpha^2}$.

273. $K_2K_a'' = \Delta\,\frac{2a}{\beta^2}, \quad K_2K_b'' = \Delta\,\frac{-2b}{\beta^2}, \quad K_2K_c'' = \Delta\,\frac{2c}{\beta^2}.$

274. $K_3K_a''' = \Delta\,\frac{2a}{\gamma^2}, \quad K_3K_b''' = \Delta\,\frac{2b}{\gamma^2}, \quad K_3K_c''' = \Delta\,\frac{-2c}{\gamma^2}.$

275. $KK_a = a\,\frac{\operatorname{sen} A \operatorname{sen} B \operatorname{sen} C}{\operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C}, \quad KK_b = b\,\frac{\operatorname{sen} A \operatorname{sen} B \operatorname{sen} C}{\operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C}.$

276. $HK = 2abc\,\frac{\sqrt{p^4 - n^4}}{m^2\sqrt{2n^4 - p^4}}.$

277. (1) $\overline{HK}^2 = R^2 + \frac{(m^2 - 6R^2)(16\Delta^2 - m^4)}{2m^4}.$

278. (1) $\overline{GK}^2 = \frac{1}{9m^4}(6m^2n^4 - m^6 - 27a^2b^2c^2).$

279. $GOK = -\frac{(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)}{12 \cdot prm^2}.$

Il primo cerchio di LEMOINE sega i lati BC, CA, AB del triangolo ABC rispettivamente nei punti: $DD'EE'FF'$.

280. $AF = \frac{b^2c}{m^2}, \quad BD = \frac{c^2a}{m^2}, \quad CE = \frac{a^2b}{m^2}.$

281. $AE = b\,\frac{b^2 + c^2}{m^2}, \quad BF = c\,\frac{a^2 + c^2}{m^2}, \quad CD = a\,\frac{a^2 + b^2}{m^2}.$

282. $DF = \frac{c}{m^2}\sqrt{n^4}, \quad FE = \frac{b}{m^2}\sqrt{n^4}, \quad ED = \frac{a}{m^2}\sqrt{n^4}.$

283. $AE' = \frac{bc^2}{m^2}, \quad BF' = \frac{ca^2}{m^2}, \quad CD' = \frac{ab^2}{m^2}.$

284. $AF' = c\,\frac{b^2 + c^2}{m^2}, \quad BD' = a\,\frac{a^2 + c^2}{m^2}, \quad CE' = b\,\frac{a^2 + b^2}{m^2}.$

285. $DD' = \frac{a^3}{m^2}, \quad EE' = \frac{b^3}{m^2}, \quad FF' = \frac{c^3}{m^2}.$

Distanza fra la retta di LEMOINE e la retta di BROCARD:

286. $d = \Delta\,\frac{8Rm^2}{n^4\sqrt{m^4 - 3n^4}}.$

Distanza di O dalla retta di LEMOINE:

287. $d = \frac{R^2m^4}{4(m^4 - 3n^4)}.$

Distanza di K dalla retta di LEMOINE:

288. $d = \frac{24R\Delta^2}{m^2\sqrt{m^4 - 3n^4}}.$

(1) LEMOINE, Congresso di Marsiglia.

Punti di Brocard, di Nagel, di Gergonne.

289. $\Omega A = \dfrac{b^2c}{n^2}, \quad \Omega B = \dfrac{c^2a}{n^2}, \quad \Omega C = \dfrac{a^2b}{n^2}.$

290. $\Omega' A = \dfrac{c^2b}{n^2}, \quad \Omega' B = \dfrac{a^2c}{n^2}, \quad \Omega' C = \dfrac{b^2a}{n^2}.$

291. $\Omega\Omega' = \dfrac{abc}{n^4}\sqrt{q^4 - n^4}.$

292. $\Omega O = \Omega' O = R^2 \dfrac{[p^2 - r(4R + r)]^2 - 12\Delta^2}{[p^2 - r(4R + r)]^2 + 4\Delta^2}.$

293. $\Omega O = \Omega' O = R^2 \dfrac{[p^2 - r(r_a + r_b + r_c)]^2 - 12\Delta^2}{[p^2 - r(r_a + r_b + r_c)]^2 + 4\Delta^2}.$

294. $\overline{\Omega I}^2 = 2Rr\left[\dfrac{a^3c + b^3a + a^3b}{[p^2 - r(4R + r)]^2 - 4\Delta^2} - 1\right].$

295. $\overline{\Omega' I}^2 = 2Rr\left[\dfrac{a^3b + b^3c + c^3a}{[p^2 - r(4R + r)]^2 - 4\Delta^2} - 1\right].$

296. $\Omega I = \dfrac{2Rr}{n^4}\left[a^2b(a - b) + b^2c(b - c) + c^2a(c - a)\right]$ (Catalan).

297. $\overline{\Omega O}^2 = \Omega' O = \dfrac{R^2}{n^4}(q^4 - n^4).$

298. $\overline{\Omega O}^2 = \overline{\Omega' O}^2 = R^2 - B\Omega \cdot C\Omega'.$

299. $\Omega\Omega' = \dfrac{abc}{n^4}\sqrt{q^4 - n^4}.$ [1]

300. $\Omega\Omega' = O\Omega\,\dfrac{4m^2}{n^2}.$

301. $\Omega I + \Omega' I = 8\dfrac{Rr^2}{n^4}\left[p^2(2R - r) - r(4R + r)^2\right].$

Distanze di Ω e Ω' dai tre lati di ABC:

302. (Ω) $d_a = 2\Delta\dfrac{abc}{n^4}\cdot\dfrac{b}{c}, \quad d_b = 2\Delta\dfrac{abc}{n^4}\cdot\dfrac{c}{a}, \quad d_c = 2\Delta\dfrac{abc}{n^4}\cdot\dfrac{a}{b}.$

303. (Ω') $d_a = 2\Omega\dfrac{abc}{n^4}\cdot\dfrac{c}{b}, \quad d_b = 2\Delta\dfrac{abc}{n^4}\cdot\dfrac{a}{c}, \quad d_c = 2\Delta\dfrac{abc}{n^4}\cdot\dfrac{b}{a}.$

Distanze di Z (punto medio del diametro di Brocard OK) dai tre lati di ABC:

304. $d_a = 2\Delta a\dfrac{2n^4 - a^2m^2}{m^2(2n^4 - q^4)}, \quad d_b = 2\Delta b\dfrac{2n^4 - b^2m^2}{m^2(2n^4 - q^4)}, \quad d_c = 2\Delta c\dfrac{2n^4 - c^2m^2}{m^2(2n^4 - q^4)}.$

---

[1] Il *massimo* di $\Omega\Omega'$ è la metà del raggio del circoncerchio del triangolo fondamentale.

Distanze di Z' dai lati di ABC:

305. $d_a = \dfrac{2\Delta}{3an^4(2n^4-q^4)}(2n^4a^2m^2 + m^4b^2c^2 - 2n^4q^4).$

306. $d_b = \dfrac{2\Delta}{3bn^4(2n^4-q^4)}(2n^4b^2m^2 + m^4a^2c^2 - 2n^4q^4).$

307. $d_c = \dfrac{2\Delta}{3cn^4(2n^4-q^4)}(2n^4c^2m^2 + m^4a^2b^2 - 2n^4q^4).$

Distanze di S, punto medio di $\Omega\Omega'$ dai lati di ABC:

308. $d_a = \dfrac{\Delta}{n^4}a(b^2+c^2), \quad d_b = \dfrac{\Delta}{n^4}b(a^2+c^2), \quad d_c = \dfrac{\Delta}{n^4}c(a^2+b^2).$

e pel suo corrispondente S' si ha:

309. $d_a = \dfrac{2\Delta}{a(q^4+3n^4)}(m^2a^2+b^2c^2), \quad d_b = \dfrac{2\Delta}{b(q^4+3n^4)}(m^2b^2+a^2c^2),$

$d_c = \dfrac{2\Delta}{c(q^4+3n^4)}(m^2c^2+a^2b^2).$

Distanza di D_1 centro d'omologia di ABC e del primo triangolo di Brocard $A_1B_1C_1$ dai lati di ABC (queste distanze sono inversamente proporzionali ai cubi dei lati corrispondenti):

310. $d_a = 2\Delta\dfrac{a^2b^2c^2}{n^4}\cdot\dfrac{1}{a^3}, \quad d_b = 2\Delta\dfrac{a^2b^2c^2}{n^4}\cdot\dfrac{1}{b^3}, \quad d_c = 2\Delta\dfrac{a^2b^2c^2}{n^4}\cdot\dfrac{1}{c^3}.$

Pel punto corrispondente D', polo di $\Omega\Omega'$ si ha:

311. $d_a = \dfrac{2\Delta}{q^4}\cdot a^3, \quad d_b = \dfrac{2\Delta}{q^4}\cdot b^3, \quad d_c = \dfrac{2\Delta}{q^4}\cdot c^3.$

312. $\overline{SH}^2 = 9R^2 - \dfrac{m^2}{2} + \dfrac{R^2m^4}{4n^4}(m^4-9n^4).$ (Lemoine).

313. $OK = \dfrac{2abc}{m^2}\cdot\dfrac{\sqrt{q^4-n^4}}{\sqrt{2n^4-q^4}}.$

314. $A_1B_1 = c\dfrac{\sqrt{q^4-n^4}}{m^2}$, ecc.

Confrontando questo valore con quello dato dalla 313 si ha il rapporto di similitudine dei triangoli $A_1B_1C_1$ e ABC, cioè:

315. $\dfrac{A_1B_1}{a} = \dfrac{D}{OK} = \dfrac{2}{\sqrt{1-3\operatorname{tg}^2\omega}} = \dfrac{m^2}{\sqrt{q^4-n^4}}.$

316. $\overline{H\Gamma}^2 = 4R^2 - 8Rp^2\cdot\dfrac{2R-r}{(4R+r)^2}.$

317. $\overline{H\Gamma}^2 = 4R^2 - 8Rp^2 \cdot \dfrac{2R-r}{(r_a+r_b+r_c)^2}$.

318. $\overline{O\Gamma}^2 = R^2 - 4p\Delta\,\dfrac{R+r}{(4R+r)^2}$.

319. $\overline{O\Gamma}^2 = R^2 - 4p\Delta\,\dfrac{R+r}{(r_a+r_b+r_c)^2}$.

320. $\overline{I\Gamma}^2 = r^2 - \dfrac{3\Delta^2}{(4R+r)^2}$.

321. $\overline{I\Gamma}^2 = r^2 - \dfrac{3\Delta^2}{(r_a+r_b+r_c)^2}$.

322. $\overline{OJ}^2 = R - 2r$.

323. $\overline{OJ}^2 = p^2 + 5r^2 - 16Rr$.

324. $\overline{I\Gamma}^2 = 16R\left[\dfrac{p^2(R+r)}{(4R+r)^2} - r\right]$.

325. $OJ\Gamma = -(b-c)(c-a)(a-b)\,\dfrac{R+r}{8Rr+2r^2}$.

326. $\overline{\Gamma H}^2 = 4R^2 - \dfrac{8p^2R}{(4R+r)^2}(2R-r)$.

Distanze di J dai lati di ABC:

327. $d_a = \dfrac{2\Delta}{ap}(p-a) \quad d_b = \dfrac{2\Delta}{bp}(p-b) \quad d_c = \dfrac{2\Delta}{cp}(p-c)$.

328. $d_a = \dfrac{r}{a}(p-a) \quad d_b = \dfrac{r}{b}(p-b) \quad d_c = \dfrac{r}{c}(p-c)$.

329. $d_a = \dfrac{h_a}{p}(p-a) \quad d_b = \dfrac{h_b}{p}(p-b) \quad d_c = \dfrac{h_c}{p}(p-c)$.

Distanze di Γ dai tre lati di ABC:

330. $d_a = \dfrac{2\Delta^3}{p(t^2-p^2)} \cdot \dfrac{1}{a(p-a)}$, ecc.

Distanze dei punti di JÉRABECK dai tre lati di ABC:

331. (J_5) $d_a = 2\Delta\,\dfrac{b}{t^2}, \quad d_b = 2\Delta\,\dfrac{a}{t^2}, \quad d_c = 2\Delta\,\dfrac{a}{t^2}$.

332. (J_p) $d_a = 2\Delta\,\dfrac{c}{t^2}, \quad d_b = 2\Delta\,\dfrac{a}{t^2}, \quad d_c = 2\Delta\,\dfrac{b}{t^2}$.

Distanze del baricentro del perimetro di ABC dai lati di questo:

333. $d_a = \Delta\,\dfrac{2p-a}{2ap}, \quad d_b = \Delta\,\dfrac{2p-b}{2bp}, \quad d_c = \Delta\,\dfrac{2p-c}{2cp}$.

Distanze di I_0, reciproco dell'incentro dai tre lati di ABC:

334. $d_a = 2\Delta \frac{abc}{t^2} \cdot \frac{1}{a^2}, \quad d_b = 2\Delta \frac{abc}{t^2} \cdot \frac{1}{b^2}, \quad d_c = 2\Delta \frac{abc}{t^2} \cdot \frac{1}{c^2}.$

Equazione della retta [1] $A\Gamma$:

335. $\frac{\beta}{\gamma} = \frac{I'B'}{I'C'},$

se poi osserviamo chè è: $\alpha \cos^2 \frac{A}{2} = \beta \cos^2 \frac{B}{2} = \gamma \cos^2 \frac{C}{2}$, essendo: $a\alpha + b\beta + c\gamma = 2\Delta$, si deduce per α_a, ad esempio, cioè per distanza di Γ da a:

336. $d_a = \frac{4\Delta^3}{abc\,(2p^2 - m^2)} \cdot \frac{1}{\cos^2 \frac{A}{2}}.$

Distanze del punto di TARRY N dai lati di ABC:

337. $d_a = 2\Delta \frac{q^4 n^4 - q^4 a^4 - 2 b^2 c^2 n^4}{a\,(q^4 - n^4)(2n^4 - q^4)}$, ecc.

Distanze del punto di STEINER S dai tre lati di ABC:

338. $d_a = 2\Delta \frac{a^4 - n^4 + 2b^2c^2}{a(q^4 - n^4)}$, ecc.

Espressioni del raggio ρ del cerchio di SCHOUTE:

339. $\rho = R^2 \frac{\operatorname{sen}^2 \omega\,(1 - 4\operatorname{sen}^2 \omega_1)}{\operatorname{sen}^2 (\omega + \omega_1)}.$

340. $\rho^2 = 4R^2 \frac{\operatorname{sen}^2 \omega \operatorname{sen}(30^\circ - \omega_1) \operatorname{sen}(30^\circ + \omega_1)}{\operatorname{sen}^2 (\omega + \omega_1)}.$

341. $\rho = \frac{R\sqrt{\lambda^2 - 3}}{\lambda + \operatorname{cotg} \omega}.$

Angolo di Brocard, ecc.

342. $\operatorname{cotg} \omega = \operatorname{cotg} A + \operatorname{cotg} B + \operatorname{cotg} C.$

343. $\operatorname{cotg} \omega = \frac{a^2 + b^2 + c^2}{4\Delta}.$

344. $\operatorname{cotg} \omega = \frac{p^2 - r(r_a + r_b + r_c)}{2\Delta}.$

345. $\operatorname{cotg} \omega = \frac{p^2 - r(4R + r)}{2\Delta}.$

---

[1] I' proiezioni di I su BC; B', C' proiezioni di I' su AC e AB.

346. $\operatorname{cotg}\omega = \frac{1}{2\Delta}\left[(p-a)^2 + r_a(4R - r_a)\right]$.

347. $\operatorname{sen}\omega = \frac{2\Delta}{\sqrt{4\Delta^2 + \left(p^2 - r(4R + r_a)\right)^2}}$.

348. $\operatorname{sen}\omega = \frac{2\Delta}{\sqrt{4\Delta^2 + \left((p-a)^2 + r_a(4R + r)\right)^2}}$.

349. $\cos\omega = \frac{\Delta}{\sqrt{4\Delta^2 + \left(p^2 - r(4R + r)\right)^2}}$.

350. $\cos\omega = \frac{\Delta}{\sqrt{4\Delta^2 + \left((p-a)^2 + r_a(4R - r_a)\right)^2}}$.

351. $\operatorname{sen}^3\omega = \operatorname{sen}(A-\omega)\operatorname{sen}(B-\omega)\operatorname{sen}(C-\omega)$.

352. $\operatorname{cosec}^2\omega = \operatorname{cosec}^2 A + \operatorname{cosec}^2 B + \operatorname{cosec}^2 C$. (NEUBERG).

353. $\operatorname{sen} A\cos(A+\omega) + \operatorname{sen} B\cos(B+\omega) + \operatorname{sen} C\cos(C+\omega) = o$. (NEUBERG).

354. $\operatorname{cotg} 2\omega = \frac{\operatorname{sen}^4 A + \operatorname{sen}^4 B + \operatorname{sen}^4 C}{2\operatorname{sen} A\operatorname{sen} B\operatorname{sen} C(\operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C)}$.

355. $4\frac{\operatorname{cotg}\omega}{\Delta} = \frac{1}{r^2} + \frac{1}{r_a^2} + \frac{1}{r_b^2} + \frac{1}{r_c^2}$.

356. $\operatorname{cotg}\omega = \frac{\operatorname{sen}^2 A + \operatorname{sen} B\operatorname{sen} C\cos A}{\operatorname{sen} A\operatorname{sen} B\operatorname{sen} C}$.

357. $\operatorname{tg}\omega = \frac{\sqrt{2n^4 - p^4}}{m^2} = \frac{2abc}{Dm^2}$.

358. $\operatorname{tg} 2\omega = \frac{m^2}{p^4}\sqrt{2n^4 - p^4} = \frac{m^4}{p^4}\operatorname{tg}\omega$.

359. $\operatorname{tg} 3\omega = \frac{p^4 + n^4}{m^3(p^4 - n^4)}\sqrt{2n^4 - p^4}$.

360. $\operatorname{tg} 4\omega = \frac{m^2p^4}{p^8 - 2n^8}\sqrt{2n^4 - p^4}$.

361. $\operatorname{tg}\omega = \frac{\operatorname{sen} 2A + \operatorname{sen} 2B + \operatorname{sen} 2C}{2(\operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C)}$.

362. $\operatorname{tg}\omega = \frac{a\cos A + b\cos B + c\cos C}{a\operatorname{sen} A + b\operatorname{sen} B + c\operatorname{sen} C}$.

363. $\frac{\Omega\Omega'}{\operatorname{sen}\omega\sqrt{1 - 4\operatorname{sen}^2\omega}} = D = \frac{a}{\operatorname{sen} A} = \frac{b}{\operatorname{sen} B} = \frac{c}{\operatorname{sen} C}$.

364. $D = \frac{\Omega\Omega'(1 + \operatorname{tg}^2\omega)}{\operatorname{tg}\omega\sqrt{1 - 3\operatorname{tg}^2\omega}}$.

365. $D = \frac{2\Omega\Omega'}{\operatorname{sen} 2\omega\sqrt{1 - 3\operatorname{tg}^2\omega}}$.

Questa relazione mostra che ω ha un massimo: $\frac{\pi}{6} = 30°$ con $\operatorname{sen}\omega = \frac{1}{2}$ e $\operatorname{tg}\omega = \frac{1}{\sqrt{3}}$.

366. $\dfrac{1}{r\operatorname{cotg}\omega} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} - \dfrac{1}{D}$.

Relazioni angolari.

367. $\cos A + \cos B + \cos C = 1 + 4\operatorname{sen}\dfrac{A}{2}\operatorname{sen}\dfrac{B}{2}\operatorname{sen}\dfrac{C}{2} = 1 + \dfrac{r}{R}$.

368. $\cos A - \cos B - \cos C = 1 - 4\operatorname{sen}\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2} = 1 - \dfrac{r_a}{R}$.

369. $\operatorname{sen}^2\dfrac{A}{2} + \operatorname{sen}^2\dfrac{B}{2} - \operatorname{sen}^2\dfrac{C}{2} = 1 - \dfrac{r_c}{2R} = 1 - \dfrac{r_b}{2R}$.

370. $\cos^2\dfrac{A}{2} + \cos^2\dfrac{B}{2} - \cos^2\dfrac{C}{2} = \dfrac{r_c}{2R} = \dfrac{r_b}{2R}$.

371. $\operatorname{cotg}\dfrac{A}{2}\operatorname{cotg}\dfrac{B}{2}\operatorname{cotg}\dfrac{C}{2} = \dfrac{p}{r}$.

372. $\operatorname{cotg}\dfrac{A}{2}\operatorname{tg}\dfrac{B}{2}\operatorname{tg}\dfrac{C}{2} = \dfrac{p-a}{r_a}$.

373. $\operatorname{sen}^2\dfrac{A}{2} = \dfrac{r_a - r}{4r}$, $\operatorname{sen}^2\dfrac{B}{2} = \dfrac{r_b - r}{4r}$, $\operatorname{sen}^2\dfrac{C}{2} = \dfrac{r_c - r}{4R}$.

374. $\cos^2\dfrac{B}{2} = \dfrac{r_a + r_c}{4R}$, $\cos^2\dfrac{C}{2} = \dfrac{r_a + r_b}{4R}$, $\cos^2\dfrac{A}{2} = \dfrac{r_b + r_c}{4R}$.

375. $\cos B\cos C - \cos C\cos A - \cos A\cos B = \dfrac{(p-a)^2 - r_a^2}{4R^2} - 1$.

376. $b\operatorname{cotg}B + c\operatorname{cotg}C - a\operatorname{cotg}A = 2(r_a - R)$.

377. $\cos A = \dfrac{2R + r - r_a}{2R}$.

378. $bc\cos A = r_b r_c - r r_a$.

379. $\dfrac{a^2}{\operatorname{tg}B} + \dfrac{b^2}{\operatorname{tg}C} + \dfrac{c^2}{\operatorname{tg}A} = \dfrac{a^4 + b^4 + c^4}{4\Delta}$.

380. $\dfrac{a^2}{\operatorname{tg}C} + \dfrac{b^2}{\operatorname{tg}A} + \dfrac{c^2}{\operatorname{tg}B} = \dfrac{a^4 + b^4 + c^4}{4\Delta}$.

381. $a\cos A + b\cos B - c\cos C = 2c\cos A\cos B$.

382. $b^2\cos C + c^2\cos B = \dfrac{a}{2}(a^2 - 4bc\cos B\cos C)$.

383. $\operatorname{sen}2A + \operatorname{sen}2B + \operatorname{sen}2C = \dfrac{2\Delta}{R^2}$.

384. $\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \dfrac{2\Delta}{p^2 - (2R + r)^2}$.

385. $\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \operatorname{tg} A \operatorname{tg} B \operatorname{tg} C$.

386. $\operatorname{tg} A + \operatorname{tg} B + \operatorname{tg} C = \dfrac{2\Delta}{(p-a)^2 - (2R - r_a)^2}$.

387. $\dfrac{a^2}{\operatorname{tg} A} + \dfrac{b^2}{\operatorname{tg} B} + \dfrac{c^2}{\operatorname{tg} C} = 4\Delta$.

388. $a^2 \cos A + b^2 \cos B + c^2 \cos C = \dfrac{r}{R}\left[3p^2 - (4R + r)(2R + r)\right]$.

389. $\cos A = \dfrac{r_b r_c - r r_a}{r_b r_c + r r_a}$.

390. $\operatorname{sen}^2 \dfrac{A}{2} = \dfrac{r_a - r}{4R}$.

391. $\operatorname{cotg} \dfrac{A}{2} = \dfrac{\operatorname{sen} B + \operatorname{sen} C}{\cos B + \cos C}$.

392. $\operatorname{sen} A = \dfrac{1}{2bc}\sqrt{2n^4 - (a^4 + b^4 + c^4)}$.

393. $\operatorname{sen} A = \dfrac{2}{bc}\sqrt{p(p-a)(p-b)(p-c)}$.

394. $\operatorname{sen} \dfrac{A}{2} = \sqrt{\dfrac{(p-b)(p-c)}{bc}}$.

395. $\cos \dfrac{A}{2} = \sqrt{\dfrac{p(p-a)}{bc}}$.

396. $\operatorname{tg} \dfrac{A}{2} = \sqrt{\dfrac{(p-b)(p-c)}{p(p-a)}}$.

397. $a^2 - b^2 \cos C - c^2 \cos B = bc(\cos B + \cos C) - 2a(r - a)$.

398. $\cos B + \cos C = \dfrac{r + r_a}{2R}$.

399. $\cos B - \cos C = \dfrac{r_c - r_b}{2R}$.

400. $\operatorname{sen} A + \operatorname{sen} B + \operatorname{sen} C = \dfrac{p}{R}$.

401. $\cos A + \cos B + \cos C = 1 + \dfrac{r}{R}$.

402. $\operatorname{sen} A \operatorname{sen} B \operatorname{sen} C = \dfrac{pr}{2R^2}$.

403. $\cos A \cos B \cos C = \dfrac{p^2 - (2R + r)^2}{4R^2}$.

404. $\operatorname{sen} \dfrac{A}{2} \operatorname{sen} \dfrac{B}{2} \operatorname{sen} \dfrac{C}{2} = \dfrac{r}{4R}$.

405. $\operatorname{tg}\frac{A}{2}+\operatorname{tg}\frac{B}{2}+\operatorname{tg}\frac{C}{2}=\frac{4R+r}{p}$.

406. $\frac{\operatorname{sen}A+\operatorname{sen}B+\operatorname{sen}C}{\operatorname{sen}A\operatorname{sen}B\operatorname{sen}C}=\frac{R}{r}$.

407. $\operatorname{tg}\frac{A}{2}(p-a)=r$.

408. $p\operatorname{tg}\frac{A}{2}=r_a$.

409. $2R(h_a\cos A+h_b\cos B+h_c\cos C)=a^2+b^2+c^2$.

410. $4R\operatorname{sen}\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=r_a$.

411. $\operatorname{sen}^2\frac{A}{2}+\operatorname{sen}^2\frac{B}{2}+\operatorname{sen}^2\frac{C}{2}=1-2\operatorname{sen}\frac{A}{2}\operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}$.

412. $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}+\cos^2\frac{C}{2}=2\left(1+\operatorname{sen}\frac{A}{2}\operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}\right)$.

413. $\operatorname{sen}A\operatorname{sen}B+\operatorname{sen}B\operatorname{sen}C+\operatorname{sen}C\operatorname{sen}A=\frac{p^2+r^2+4Rr}{4R^2}$.

414. $\cos A\cos B+\cos B\cos C+\cos C\cos A=\frac{p^2+r^2-4R^2}{4R^2}$.

415. $a\cos A+b\cos B+c\cos C=4R\operatorname{sen}A\operatorname{sen}B\operatorname{sen}C$.

416. $\operatorname{sen}^2A+\operatorname{sen}B\operatorname{sen}C\cos A=\operatorname{sen}^2B+\operatorname{sen}A\operatorname{sen}C\cos B=\operatorname{sen}^2C+\operatorname{sen}A\operatorname{sen}B\cos C$.

417. $\frac{a\operatorname{sen}A+b\operatorname{sen}B+c\operatorname{sen}C}{a\cos A+b\cos B+c\cos C}=R\frac{m^2}{abc}$.

418. $\operatorname{sen}A+\operatorname{sen}B+\operatorname{sen}C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

419. $\operatorname{cotg}A+\operatorname{cotg}B+\operatorname{cotg}C=\operatorname{cotg}A\operatorname{cotg}B\operatorname{cotg}C+\operatorname{cosec}A\operatorname{cosec}B\operatorname{cosec}C$.

420. $4\operatorname{sen}A\operatorname{sen}B\operatorname{sen}C=\operatorname{sen}2A+\operatorname{sen}2B+\operatorname{sen}2C$.

421. $2\operatorname{sen}A\operatorname{sen}B\operatorname{sen}C=\operatorname{sen}^2A+\operatorname{sen}^2B+\operatorname{sen}^2C$.

422. $(\operatorname{sen}A+\cos A)(\operatorname{sen}B+\cos B)(\operatorname{sen}C+\cos C)=2\operatorname{sen}A\operatorname{sen}B\operatorname{sen}C+2\cos A\cos B\cos C+1$.

423. $\operatorname{sen}^2\frac{A}{2}+\operatorname{sen}^2\frac{B}{2}+\operatorname{sen}^2\frac{C}{2}=1-2\operatorname{sen}\frac{A}{2}\operatorname{sen}\frac{B}{2}\operatorname{sen}\frac{C}{2}$.

424. $\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}=4\cos\frac{A+B}{4}\cos\frac{B+C}{4}\cos\frac{C+A}{4}$.

425. $\operatorname{cotg}\frac{C}{2}=\frac{\operatorname{sen}A+\operatorname{sen}B}{\cos A+\cos B}$.

426. $\cos 2A+\cos 2B+\cos 2C=-(4\cos A\cos B\cos C+1)$.

427. $\cos 2A+\cos 2B-\cos 2C=4\operatorname{sen}A\operatorname{sen}B\cos C-1$.

428. $\cos 4A+\cos 4B+\cos 4C=4\cos 2A\cos 2B\cos 2C-1$.

429. $\operatorname{sen}^2 2A+\operatorname{sen}^2 2B+\operatorname{sen}^2 2C=-2(\cos 2A\cos 2B\cos 2C-1)$.

430. $4R^2 \cos A \cos B \cos C = p^2 - (2R + r)^2$.

431. $b\left(\operatorname{cotg}\frac{B}{2} - \operatorname{tg}\frac{A}{2}\right) + c\left(\operatorname{cotg}\frac{C}{2} - \operatorname{tg}\frac{A}{2}\right) = 4R$.

432. $4R^2 \cos A \cos B \cos C = p^2 - (2R + r)^2$.

433. $bc \cos A + ac \cos B - ab \cos C = 2\Delta\,(\operatorname{cotg} A + \operatorname{cotg} B - \operatorname{cotg} C)$.

434. $\operatorname{sen} A + \operatorname{sen} B + \operatorname{sen} C = \frac{\operatorname{tg} A}{\operatorname{tg} B} + \frac{\operatorname{tg} B}{\operatorname{tg} A} + \frac{\operatorname{tg} A}{\operatorname{tg} C} + \frac{\operatorname{tg} C}{\operatorname{tg} A} + \frac{\operatorname{tg} B}{\operatorname{tg} C} + \frac{\operatorname{tg} C}{\operatorname{tg} B} + 2$.

435. $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$.

436. $\operatorname{sen}^3 A + \operatorname{sen}^3 B + \operatorname{sen}^3 C = 3 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2} + \cos^3\frac{A}{2} + \cos^3\frac{B}{2} + \cos^3\frac{C}{2}$.

437. $\frac{\operatorname{tg} A}{\operatorname{tg} B} = \frac{a^2 + c^2 - b^2}{b^2 + c^2 - a^2}$.

438. $\frac{\operatorname{sen}(A - B)}{\operatorname{sen}(A + B)} = \frac{a^2 - b^2}{c^2}$.

439. $\Sigma \cos B \cos C = \frac{p^2 + r^2}{4R^2} - 1$.

440. $\Sigma \cos^2 A = \frac{1}{2R^2}\left[2R^2 + (2R + r)^2 - p^2\right]$.

441. $\Sigma\, a \cos^3 A = \frac{\Delta}{R^3}\left[R^2 + (2R + r)^2 - p^2\right]$.

442. $\Sigma \frac{a^2}{\operatorname{tg} A} = 4\Delta$.

Area del Triangolo.

443. $\Delta = r_a r_b \frac{r_c - r}{c}$.

444. $\Delta = p r_a \left(1 - \frac{2r}{h_a}\right)$.

445. $\Delta = r(p - a)\left(1 + \frac{2r_a}{h_a}\right)$.

446. $\Delta = \frac{1}{a}\left(r r_a (r_b + r_c)\right)$.

447. $\Delta = \frac{1}{c}\left(r_a r_b (r_c - r)\right)$.

448. $\Delta = \frac{r \cdot r_a \cdot r_b \cdot r_c}{p}$.

449. $\Delta = \frac{r \cdot r_b \cdot r_c}{p - a} = \frac{r \cdot r_a \cdot r_c}{p - b} = \frac{r \cdot r_a \cdot r_b}{p - c}$.

450. $\Delta = \sqrt{p\,(p-a)\,(p-b)\,(p-c)}$.

451. $\Delta = \dfrac{BC \,.\, AX}{2}$.

452. $\Delta = \dfrac{1}{2}\sqrt[3]{abch_ah_bh_c}$.

453. $\Delta = \dfrac{l_al_bl_c\,(a+b)\,(b+c)\,(c+a)}{8\,abcp}$.

454. $\Delta = \dfrac{1}{R}\,abc$.

455. $\Delta^2_0 = R\,(r_b+r_c)\,(r_c+r_a)\,(r_a+r_b)$.

456. $\Delta^2_1 = R\,(r_b+r_c)\,(r_b-r)\,(r_c-r)$.

457. $\Delta^2_2 = R\,(r_c+r_a)\,(r_c-r)\,(r_a-r)$.

458. $\Delta^2_3 = R\,(r_a+r_b)\,(r_a-r)\,(r_b-r)$.

459. $\Delta^2 = \Delta_0\Delta_t$.

460. $\Delta = \dfrac{1}{3}\sqrt{(m_a+m_b+m_c)\,(m_a+m_b-m_c)\,(m_b+m_c-m_a)\,(m_c+m_a-m_b)}$.

461. $\dfrac{2}{\Delta} = \dfrac{1}{\Delta_1} + \dfrac{1}{\Delta_2} + \dfrac{1}{\Delta_3} - \dfrac{1}{\Delta_0}$.

462. $\Delta_0 r = \Delta_1 r_a = \Delta_2 r_b = \Delta_3 r_c$.

463. $\Delta = \dfrac{1}{2}\,R\,(XY + YZ + ZX)$.

464. $\Delta = \dfrac{r^2\,(r_a-r_b)\,(r_a-r_c)\,(r_b-r_c)}{(a-b)\,(b-c)\,(c-a)}$.

465. $\Delta = \dfrac{a-b}{\dfrac{1}{r_b} - \dfrac{1}{r_a}}$.

466. $\Delta = 2\,R^2\,\dfrac{h_ah_bh_c}{abc}$.

467. $\Delta = \sqrt{r \,.\, r_a \,.\, r_b \,.\, r_c}$.

468. $\Delta = \dfrac{r_ar_br_c}{p}$.

469. $\Delta = (a+b)\,\dfrac{rr_c}{r+r_c}$.

470. $\Delta = (a-b)\,\dfrac{r_ar_b}{r_a-r_b}$.

471. $\Delta = \dfrac{1}{4}\sqrt{8\,m_c^2\,(a^2+b^2-2\,m_c^2) - (a^2-b^2)^2}$.

472. $\Delta = \dfrac{1}{2}\,h_c\left(\sqrt{a^2-h_c^2} + \sqrt{b^2-h_c^2}\right)$.

473. $\Delta = \dfrac{2}{3}\sqrt{m_b^2m_c^2 - K_a^4}$ (essendo $2\,K_a^4 = \dfrac{9}{4}\,a^2 - (m_b^2+m_c^2)$.

474. $2\,\Delta\,a^2b^2 - \Delta\,a^4 = 16\,\Delta.$

475. $\Delta_1\,\Delta'_a = \Delta_2\,\Delta'_b = \Delta_3\,\Delta'_c.$

476. $\Delta = \frac{1}{2}\,(\Delta'_a + \Delta'_b + \Delta'_c - \Delta_t).$

477. $\frac{1}{\Delta_t} = \frac{1}{\Delta'_a} + \frac{1}{\Delta_b'} + \frac{1}{\Delta_c'}.$

478. $\Delta^3 = 4\,R^2\sqrt{\Delta_t\,\Delta_a\,\Delta_b\,\Delta_c}.$

479. $\frac{\Delta_t}{\Delta} = \frac{2\,pr^2}{abc} = \frac{r}{2\,R}.$

480. $\frac{\Delta_1}{r} = \frac{\Delta}{2\,R}.$

481. $\Delta = \frac{1}{6}\,h_a\,(\sqrt{4m_b^2 - h_a^2} + \sqrt{4\,m_c^2 - h_a^2}).$

482. $\Delta = \frac{h_b^3\sqrt{a^2 - h_b^2} - h_c^3\sqrt{a^2 - h_c^2}}{2\,(h_b^2 - h_c^2)}.$

483. $\Delta = \frac{1}{2}\sqrt{ab\,h_a h_b} = \frac{1}{2}\cdot\frac{a \pm b}{\frac{1}{h_a} \pm \frac{1}{h_b}}.$

484. $\Delta = \frac{pr_a h_a}{2\,(r_a + h_a)}.$

485. $\Delta = \frac{a\,b}{4\,R}\sqrt{2\,(a^2 + b^2 - 2\,m_c^2)}.$

486. $\Delta = h_a\,(\sqrt{b^2 - h_a^2} \pm \sqrt{m_a^2 - h_a^2}).$

487. $\Delta = \frac{ab}{8\,R^2}\,(a\sqrt{4\,R^2 - b^2} + b\sqrt{4\,R^2 - a^2}).$

488. $\Delta = (r_b + r_c)\,\frac{rr_a}{r}.$

489. $\Delta = rr_a\sqrt{\frac{4\,R - (r_a - r)}{r_a - r}}.$

490. $\Delta = r_a r_b\sqrt{\frac{\overline{I_a I_b}^2}{(r_a + r_b)^2} - 1}.$

491. $\Delta^2 = \frac{16}{9}\left[m\,(m - m_a)(m - m_b)(m - m_c)\right].$

Se per brevità poniamo:

$$m_a + m_b + m_c = 2\,q, \quad m_b + m_c - m_a = 2\,q_a, \quad m_a + m_c - m_b = 2\,q_b$$

$$m_a + m_b - m_c = 2\,q_c, \quad m_b + m_c - \frac{3}{2}\,a = 2\,q'_a, \quad m_a + m_c - \frac{3}{2}\,b = 2\,q_b$$

$$m_a + m_b - \frac{3}{2}\,c = 2\,q'_c.$$

492. $\Delta = \frac{4}{3}\sqrt{q \cdot q_a \cdot q_b \cdot q_c}$.

493. $\Delta = \frac{4}{3}\sqrt{q'_a\,(q'_a - m_b)\,(q'_a - m_c)\,(q'_a - \frac{3}{2}\,a)}$.

494. $\Delta = \frac{4}{3}\sqrt{q'_b\,(q'_b - m_c)\,(q'_b - m_a)\,(q'_b - \frac{3}{2}\,b)}$.

495. $\Delta = \frac{4}{3}\sqrt{q'_c\,(q'_c - m_b)\,(q'_c - m_a)\,(q'_c - \frac{3}{2}\,c)}$.

496. $\Delta = \frac{1}{4}\sqrt{2\,m^4 - q^4}$.

497. $\Delta = 2R^2\left[(f-2x^2)\sqrt{(g-2x^2)(1-g+2x^2)} + (g-2x^2)\sqrt{(f-2x^2)(1-f+2x^2)}\right]$

dov'è:

$$f = \frac{m_a^2 + 2\,m_b^2}{3\,R^2}, \qquad g = \frac{2\,m_a^2 + m_b^2}{3\,R^2}.$$

ed $x\,(=\operatorname{sen} C)$ è dato dall'equazione:

$$x = \sqrt{(f-2x^2)(1-g+2x_2)} + \sqrt{(g-2x^2)(1-f+2x^2)}.$$

498. $\Delta = \frac{1}{2}\,h_a^2\,\frac{\operatorname{sen} A}{\operatorname{sen} B \operatorname{sen} C}$.

499. $\Delta = \frac{bc}{2}\operatorname{sen} A$.

500. $\Delta = a^2\,\frac{\operatorname{sen} B \operatorname{sen} C}{2 \operatorname{sen} A}$.

501. $\Delta = \frac{abc^2}{2\,(a^2 - b^2)}\operatorname{sen}(A - B)$.

502. $\Delta = (a^2 - b^2)\,\frac{\operatorname{sen} A \operatorname{sen} B}{2 \operatorname{sen}(A - B)}$.

503. $\Delta = \frac{m^2}{4\,(\operatorname{cotg} A + \operatorname{cotg} B + \operatorname{cotg} C)}$.

504. $\Delta = \frac{1}{4}\,(a^2 \operatorname{sen} 2B + b^2 \operatorname{sen} 2A)$.

505. $\Delta = \frac{1}{2}\sqrt[3]{a^2b^2c^2 \operatorname{sen} A \operatorname{sen} B \operatorname{sen} C}$.

506. $\Delta = \frac{AL}{2} + (b\,c)\operatorname{sen}\frac{A}{2}$.

507. $\Delta = \frac{AL}{2}\,BC \cdot \cos\frac{B-C}{2}$.

508. $\Delta = \frac{1}{2} l_a^2 \frac{\operatorname{sen} A \cos^2 \frac{1}{2}(B-C)}{\operatorname{sen} B \operatorname{sen} C}$.

509. $\Delta = \frac{\frac{1}{2}(a^2 - b^2)}{\operatorname{cotg} A - \operatorname{cotg} B}$.

510. $\Delta = \frac{1}{2} \frac{h_a h_b}{\operatorname{sen} C}$.

511. $\Delta = \frac{1}{2} a^2 \frac{\operatorname{sen} B \operatorname{sen} C}{\operatorname{sen} A} = \frac{\frac{1}{2} a^2}{\operatorname{cotg} B + \operatorname{cotg} C}$.

512. $\Delta = \frac{1}{2} h_a^2 \frac{\operatorname{sen} A}{\operatorname{sen} B \operatorname{sen} C}$.

513. $\Delta = \frac{1}{2} h_a^2 (\operatorname{cotg} B + \operatorname{cotg} C)$.

514. $\Delta = \frac{1}{2} h_a^2 (\operatorname{sen} 2B + \operatorname{sen} 2C)$.

515. $\Delta = \frac{1}{2}(a^2 - b^2) \frac{\operatorname{sen} A \operatorname{sen} B}{\operatorname{sen}(A-B)}$.

516. $\Delta = \frac{1}{2}(a^2 \operatorname{sen} 2B + b^2 \operatorname{sen} 2A)$.

Se μ e φ sono due angoli ausiliari determinati dalle relazioni: $\cos\mu = \frac{h_a}{m_a}$, $\cos\varphi = \frac{h_a}{l_a}$, è:

517. $\Delta = \frac{h_a^2}{\cos\mu} \sqrt{\frac{\operatorname{sen} 2(\mu - \varphi)}{\operatorname{sen} 2\varphi}}$.

518. $\Delta = \frac{h_a h_b}{2 \operatorname{sen} C}$.

519. $\Delta = l_a l_b \cos\frac{1}{2}(B-C) \cos\frac{1}{2}(C-A) \cdot \frac{1}{2 \operatorname{sen} C}$.

520. $\Delta = r^2 \operatorname{cotg}\frac{A}{2} \operatorname{cotg}\frac{B}{2} \operatorname{cotg}\frac{C}{2}$.

521. $\Delta = r_a^2 \operatorname{cotg}\frac{A}{2} \operatorname{tg}\frac{B}{2} \operatorname{tg}\frac{C}{2}$.

522. $\Delta = p^2 \operatorname{tg}\frac{A}{2} \operatorname{tg}\frac{B}{2} \operatorname{tg}\frac{C}{2}$.

523. $\Delta = \frac{abc}{p} \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$.

524. $\Delta = \sqrt{abcp \operatorname{sen}\frac{A}{2} \operatorname{sen}\frac{B}{2} \operatorname{sen}\frac{C}{2}}$.

525. $\Delta = \dfrac{m_a^2 + m_b^2 + m_c^2}{3\,(\operatorname{cotg} A + \operatorname{cotg} B + \operatorname{cotg} C)}$.

526. $\Delta = R\,(a\cos B\cos C + b\cos C\cos A + c\cos A\cos B)$.

527. $\Delta = \dfrac{1}{2}\,h_a h_b \dfrac{\operatorname{tg}\frac{1}{2}A\operatorname{tg}\frac{1}{2}B\operatorname{tg}\frac{1}{2}C}{\cos A}$.

528. $\Delta = h_a h_b \dfrac{\operatorname{tg}\frac{1}{2}A\operatorname{tg}\frac{1}{2}B\operatorname{tg}\frac{1}{2}C}{\cos A\left(1-\operatorname{tg}^2\frac{1}{2}B\right)}$.

529. $\Delta = \dfrac{2\,m_a^2 \operatorname{sen} A\operatorname{sen} B\operatorname{sen} C}{2\,(\operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C) - 3\operatorname{sen}^2 A}$.

530. $\Delta = \dfrac{2\,(m_b^2 - m_a^2)}{3\,(\operatorname{cotg} A - \operatorname{cotg} B)}$.

531. $\Delta = \dfrac{1}{3}\sqrt{4\,m_b^2 m_c^2 - [9\,R^2\operatorname{sen}^2 A - (m_b^2 + m_c^2)]^2}$.

532. $\Delta = \sqrt{\dfrac{1}{2}\,R\,l_a l_b l_c \operatorname{sen}(A+\frac{1}{2}B)\operatorname{sen}(B+\frac{1}{2}C)\operatorname{sen}(C+\frac{1}{2}A)}$.

Se b_a, b_b, b_c sono i lati del triangolo determinato dalle bisettrici del triangolo ABC;

533. $\Delta = \dfrac{1}{4R}\,b_a b_b b_c \operatorname{sen}\frac{1}{2}A\operatorname{sen}\frac{1}{2}B\operatorname{sen}\frac{1}{2}C$.

534. $\Delta = 2\,R^2 \operatorname{sen} A\operatorname{sen} B\operatorname{sen} C$.

535. $\Delta = R\,a\operatorname{sen} B\operatorname{sen} C$.

536. $\Delta = 4\,R\,p\operatorname{sen}\dfrac{A}{2}\operatorname{sen}\dfrac{B}{2}\operatorname{sen}\dfrac{C}{2}$.

537. $\Delta = \dfrac{R}{2}\,(a\cos A + b\cos B + c\cos C)$.

538. $\Delta = r^2 \operatorname{cotg}\dfrac{A}{2}\operatorname{cotg}\dfrac{B}{2}\operatorname{cotg}\dfrac{C}{2}$.

539. $\Delta = r^2 \operatorname{cotg}\dfrac{A}{2} + 2\,Rr\operatorname{sen} A$.

540. $\Delta = Rr\,(\operatorname{sen} A + \operatorname{sen} B + \operatorname{sen} C)$.

541. $\Delta = \dfrac{4\,m_a^2 + 3\,a^2}{8\,(\operatorname{cotg} A + \operatorname{cotg} B + \operatorname{cotg} C)}$.

542. $\Delta = \dfrac{4\,(m_a^2 + m_b^2) - 3\,c^2}{4\,(\operatorname{cotg} A + \operatorname{cotg} B + \operatorname{cotg} C)}$.

543. $\Delta = R^2 \operatorname{sen} 2A\,(1+\cos C)$.

544. $\Delta = R\,a\operatorname{sen} B\operatorname{sen} C$.

545. $\Delta = Ra\,(\cos A + \cos B \cos C)$.

546. $\Delta = R h_a \operatorname{tg} A \operatorname{tg} B \operatorname{tg} \frac{1}{2} C$.

547. $\Delta = \dfrac{h_a\,(h_b + h_c)}{4 \operatorname{sen}\left(C + \frac{1}{2} A\right) \cos \frac{1}{2} A}$.

548. $\Delta = \frac{2}{3} R^2 \left[\operatorname{sen}^3 A \cos (B - C) + \operatorname{sen}^3 B \cos (C - A) + \operatorname{sen}^3 C \cos (A - B)\right]$.

549. $\Delta = \frac{1}{4} \sqrt{2\,r \cdot I_a I_b \cdot I_b I_c \cdot I_c I_a \operatorname{sen} A \operatorname{sen} B \operatorname{sen} C}$.

550. $\Delta = \frac{1}{2} a \operatorname{sen} B\,(a \cos B + \sqrt{b^2 - a^2 \operatorname{sen}^2 B})$.

551. $\Delta = \frac{1}{2} b \operatorname{sen} A\,(b \cos A + \sqrt{a^2 - b^2 \operatorname{sen}^2 A})$.

552. $\Delta = a\left(-\, l_a \operatorname{sen} \frac{1}{2} A + l_a \operatorname{sen} \frac{1}{2} B + l_c \operatorname{sen} \frac{1}{2} C\right)$.

553. $\Delta = 2p\left(l_a \operatorname{sen} \frac{1}{2} A + l_b \operatorname{sen} \frac{1}{2} B + l_c \operatorname{sen} \frac{1}{2} C\right)$.

554. $\Delta = r^2 \operatorname{cotg} \frac{1}{2} A \operatorname{cotg} \frac{1}{2} B \operatorname{cotg} \frac{1}{2} C = r_a \operatorname{tg} \frac{1}{2} A \operatorname{tg} \frac{1}{2} B \operatorname{tg} \frac{1}{2} C$.

555. $\Delta = p^2 \operatorname{tg} \frac{1}{2} A \operatorname{tg} \frac{1}{2} B \operatorname{tg} \frac{1}{2} C = (p - a)^2 \operatorname{tg} \frac{1}{2} A \operatorname{cotg} \frac{1}{2} B \operatorname{cotg} \frac{1}{2} C$.

556. $\Delta = r^2 \operatorname{cotg} \frac{1}{2} A + 2Rr \operatorname{sen} A = r_a^2 \operatorname{cotg} \frac{1}{2} A - 2Rr_a \operatorname{sen} A$.

557. $\Delta = r^2_b \operatorname{cotg} \frac{1}{2} A - 2Rr_b \operatorname{sen} A = r_c^2 \operatorname{cotg} \frac{1}{2} A - 2Rr_c \operatorname{sen} A$.

Se con k_a, k_b, k_c indichiamo le distanze dell'ortocentro H dai tre lati e con k'_a, k'_b, k'_c le distanze dai tre vertici del triangolo ABC, è:

558. $\Delta = \frac{1}{2} (ak_a + bk_b + ck_c)$.

559. $\Delta = \frac{1}{4} (ak'_a + bk'_b + ck'_c)$.

560. $\Delta = \frac{1}{8R} (a^2 k'_a \operatorname{cosec} A + b^2 k'_b \operatorname{cosec} B + c^2 k'_c \operatorname{cosec} C)$.

Se Δ_t è l'area del triangolo formato dai piedi delle altezze di ABC:

561 $\Delta = \dfrac{\Delta_t}{2 \cos A \cos B \cos C}$.

Se Δ_h è l'area del triangolo determinato dai piedi delle mediane, è:

562. $\Delta = 4\,\Delta_h$.

Se Δ_m è l'area del triangolo determinato dai piedi delle bisettrici interne, è:

563. $\Delta = \Delta_m \dfrac{(a+b)(b+c)(c+a)}{2\,abc}$.

Se Δ_n è l'area del triangolo formato unendo i punti di tanganza dell'incerchio:

564. $\Delta = \dfrac{2R}{r}\,\Delta_n$.

Se $x_a y_a$, $x_b y_b$, $x_c y_c$ sono le coordinate ortogonali dei tre vertici, è:

565. $\Delta = \dfrac{1}{2}\left[y_a(x_c - x_b) + y_b(x_a - x_c) + y_c(x_b - x_a)\right]$

ed in coordinati polari:

566. $\Delta = \dfrac{1}{2}\left[r_a r_b \operatorname{sen}(\theta'' - \theta') + r_b r_c \operatorname{sen}(\theta''' - \theta'') + r_c r_a \operatorname{sen}(\theta' - \theta''')\right]$.

R. PROCURA
927,067